Channel Dependent Adaptive Modulation and Coding Without Channel State Information at the Transmitter

Bradford D. Boyle, John MacLaren Walsh, and Steven Weber

Modeling & Analysis of Networks Laboratory
Drexel University
Department of Electrical and Computer Engineering
http://network.ece.drexel.edu

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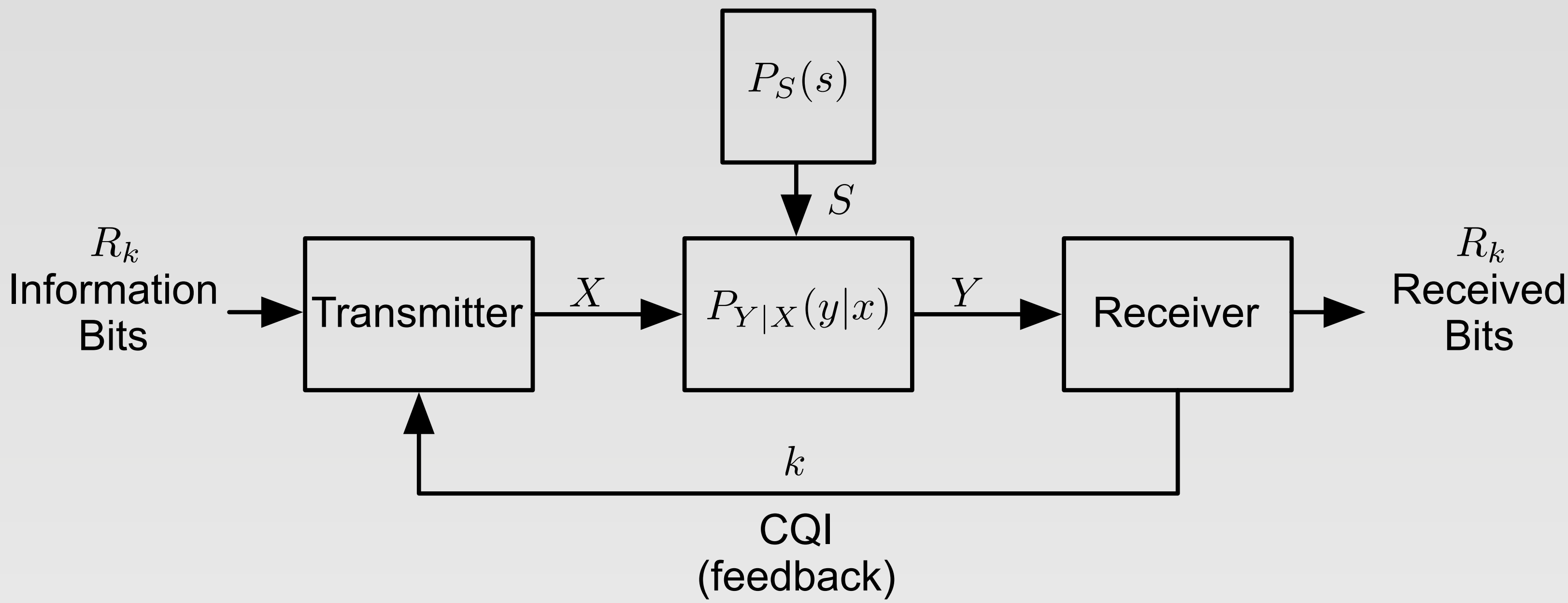
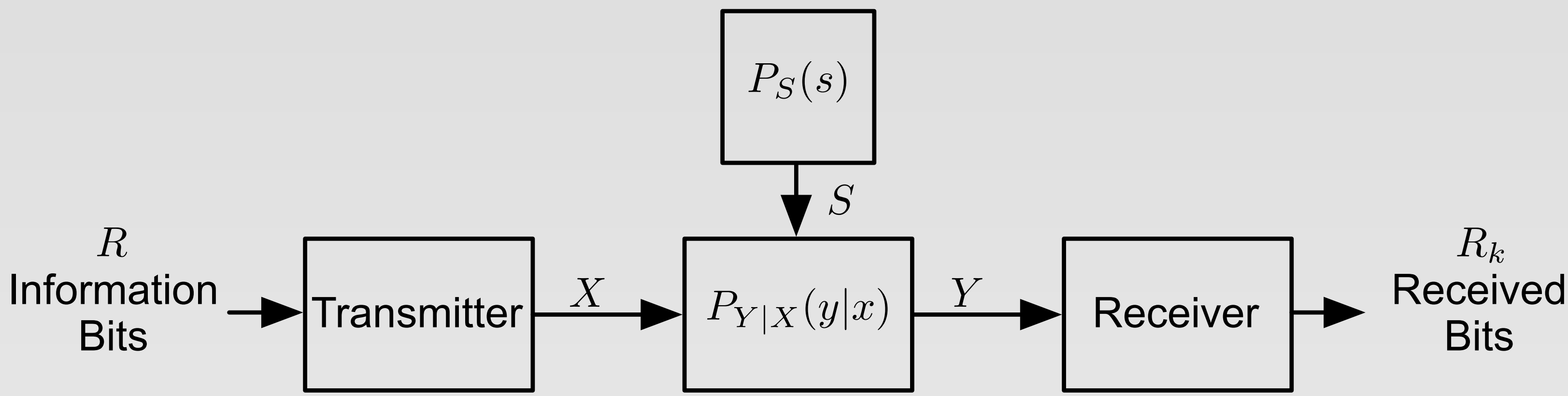
Introduction



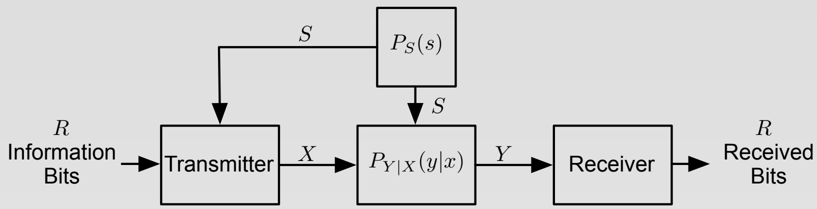
- Traditional AMC adjust the rate of information flow based on CSI feedback from a receiver
- Feedback can consume a non-trivial amount of capacity
- Omniscient transmitters would select an appropriate modulation and coding scheme without feedback

AMC without Feedback

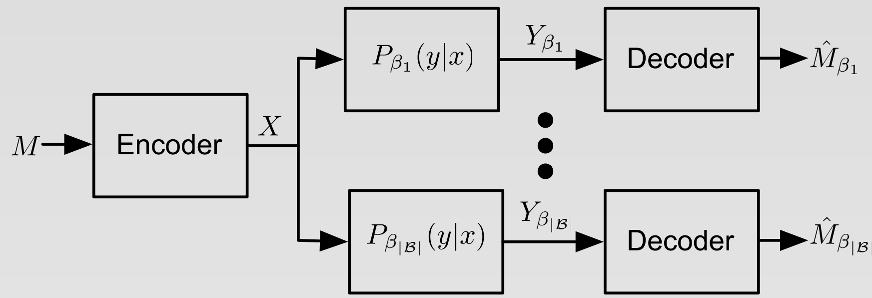


- We consider a coding strategy for conveying a variable number of information bits across a channel with an unknown SNR
- Propose two metrics for comparing its performance to an omniscient transmitter and characterize an optimal strategy
- Investigate performance scaling in the number of unknown states and the difference between the best and worst state

Related Work



Gelfand-Pinkser [1]



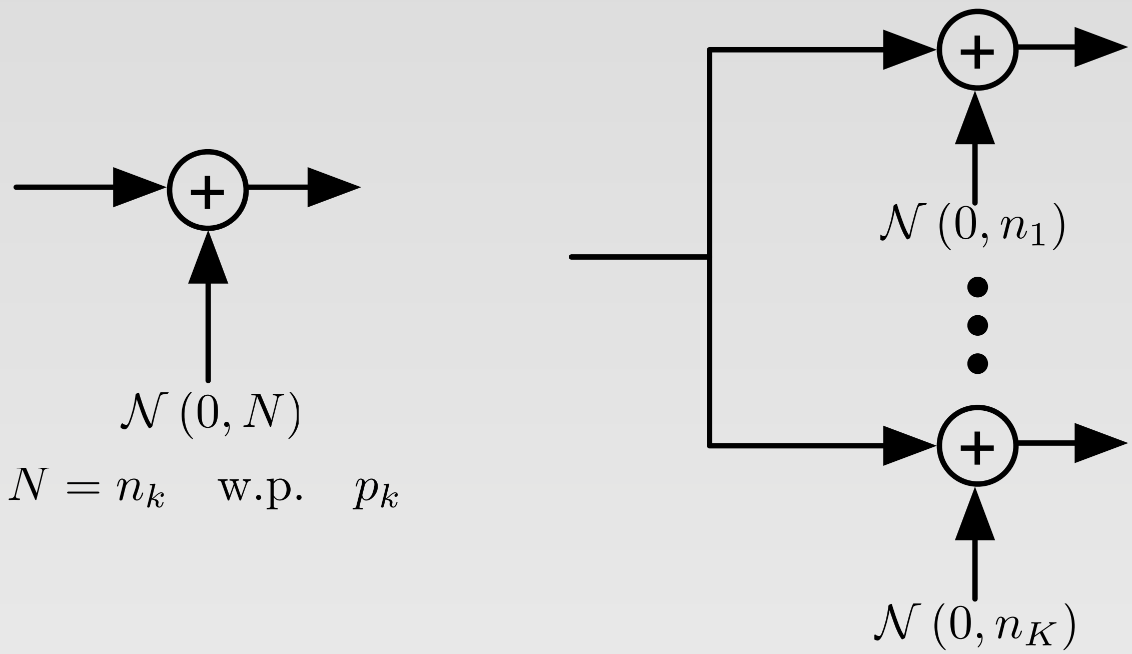
Approach to finite compound channels [2]

- Broadcast coding strategy used for Rayleigh fading AWGN channel [3]

Variable to fixed channel capacity [4]

- Variable number $L_n \leq m_n$ of bits correctly received for fixed blocklength n
- Provides an expression for channel capacity for AWGN channel with a continuous support set for SNR; our results match in the limit as $K \to \infty$ for uniformly distributed (in dB) SNR

Problem Model



- Point-to-point AWGN channel with unknown SNR drawn from a finite set of possible sets
- Model as a broadcast channel with "virtual" receivers for each possible state

$$R_k \leq \mathsf{C}\left(\frac{\alpha_k}{\sum_{i<k} \alpha_i + \gamma_k^{-1}}\right) \quad k = 1, \ldots, K \qquad [5,\ 6]$$

- If the channel is in state i, the receiver will receive $R_i + \cdots + R_K$ information bits

Gap to Omniscience

Metrics

1. Expected Capacity Loss

$$J_{\mathbb{E},|\cdot|}\left(\boldsymbol{\alpha}\right) = \sum_{k=1}^{K} p_k \left(\mathsf{C}(\gamma_k) - \sum_{i=k}^{K} \mathsf{C}\left(\frac{\alpha_i}{\sum_{j<i} \alpha_j + \gamma_i^{-1}} \right) \right)$$

or alternatively

$$J_{\mathbb{E},|\cdot|}\left(c\right) = \mathbb{E}\left[\mathsf{C}\left(\Gamma\right)\right] - \sum_{k=1}^{K} f_k \mathsf{C}\left(\frac{c_k - c_{k-1}}{c_{k-1} + \gamma_k^{-1}} \right)$$

$$f_k \triangleq \sum_{i=1}^{k} p_i = \mathbb{P}\left[\Gamma \geq \gamma_k\right] \qquad c_k = \sum_{i=1}^{I} \alpha_i$$

2. Fractional Expected Capacity Loss

$$J_{\%,\mathbb{E}}\left(\boldsymbol{\alpha}\right) = \frac{J_{\mathbb{E},|\cdot|}\left(\boldsymbol{\alpha}\right)}{\mathbb{E}\left[\mathsf{C}\left(\Gamma\right)\right]}$$

Gap to Omniscience

Optimization

The fraction of power allocated α_i for each channel state γ_i should be chosen to minimize the expected capacity loss

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{minimize}} \quad & J_{\mathbb{E},|\cdot|}(\boldsymbol{\alpha}) \\ \text{subject to} \quad & \alpha_k \geq 0 \quad k = 1, \ldots, K \\ & \sum_{k=1}^{K} \alpha_k = 1 \end{aligned}$$

which is equivalent to

$$\begin{aligned} \underset{c}{\text{maximize}} \quad & \sum_{k=1}^{K} f_k C\left(\frac{c_k - c_{k-1}}{c_{k-1} + \gamma_k^{-1}}\right) \\ \text{subject to} \quad & c_k \geq c_{k-1} \quad k = 1, \ldots, K \end{aligned}$$

Note: The optimization is *not* convex.

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General Solution

- Partial characterization given by

$$c_i^* = \frac{f_i\gamma_i - f_{i+1}\gamma_{i+1}}{\gamma_i\gamma_{i+1}(f_{i+1} - f_i)}$$

  if $c_{i-1}^* \neq c_i^* \neq c_{i+1}^*$ with $c_0^* = 0$ and $c_K^* = 1$.

- If f_i's and γ_i's are such that the above is an increasing sequence in i, then

$$c_i^* = \left[\frac{f_i\gamma_i - f_{i+1}\gamma_{i+1}}{\gamma_i\gamma_{i+1}(f_{i+1} - f_i)}\right]_0^1$$

  where $[\cdot]_0^1 = \max\{\min\{\cdot, 1\}, 0\}$.

- If $p_i = {}^1/_K$ and $\gamma_i = \delta^{K-i}\gamma_K$ with $\delta > 1$, then the above is an increasing sequence.

Gap to Omniscience

Channels with Two States

Let $K = 2$, $\gamma_1 = \delta\gamma_2$, $\delta \geq 1$, and $\alpha = \alpha_1$.

- The optimal fraction of the power to allocate for the better SNR (γ_1) is

$$\alpha^* = \left[\frac{p_1\delta - 1}{\delta\gamma_2(1 - p_1)}\right]_0^1$$

- Properties of α^*
  - $\alpha^* = 0$ for $1 \leq \delta \leq {}^1\!/_{p_1}$
  - If $p_1 < 1$ then α^* is monotone increasing in δ
  - If $p_1 \leq {}^{\gamma_2}\!/_{\gamma_2+1}$, then

$$\lim_{\delta\to\infty} \alpha^* = \frac{p_1}{\gamma_2(1 - p_1)}.$$

If $p_1 > {}^{\gamma_2}\!/_{\gamma_2+1}$, then $\alpha^* = 1$ for all $\delta \geq {}^1\!/_{(p_1-\gamma_2(1-p_1))}$

For certain parameters (p_1, γ_2), full power is not allocated for the state with the better SNR (γ_1) independent of how much better that state might be.

Gap to Omniscience

Channels with Two States

- Properties of Expected Capacity Loss
  - If $\delta = 1$, then $J_{\mathbb{E},|\cdot|} = 0$
  - $J_{\mathbb{E},|\cdot|}$ is increasing in δ
  - If $p_1 \leq \gamma_2/\gamma_2+1$,

$$\lim_{\delta \to \infty} J_{\mathbb{E},|\cdot|}(\alpha^*) = \frac{p_1}{2} \log_2 \left(\frac{\gamma_2(1-p_1)}{(1+\gamma_2)p_1} \right) - \frac{1}{2} \log_2 (1-p_1)$$

    and if $p_1 > \gamma_2/\gamma_2+1$

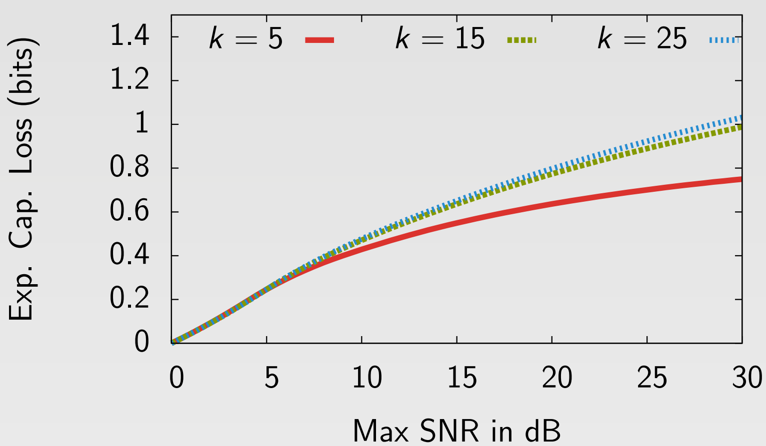
$$\lim_{\delta \to \infty} J_{\mathbb{E},|\cdot|}(\alpha^*) = \frac{1-p_1}{2} \log_2 (1+\gamma_2)$$

- Properties of Fractional Expected Capacity Loss
  - $J_{\%,|\cdot|}$ is increasing in δ for $1 \leq \delta \leq 1/p_1$
  - $J_{\%,|\cdot|}$ is non-monotonic in δ
  - $$\lim_{\delta \to \infty} J_{\%,\mathbb{E}}(\boldsymbol{\alpha}) = 0$$

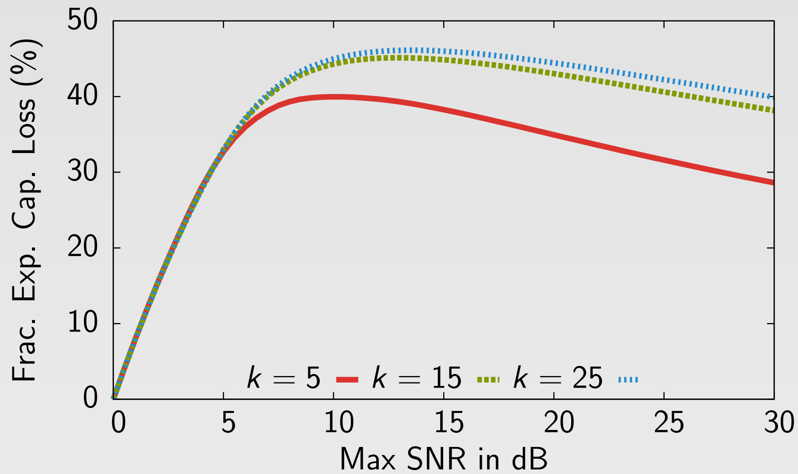
Results

K-State Channel Results

Gap to Omniscience metrics as a function of K and max SNR γ_1 with $\gamma_K = 0$ dB, $p_i = {}^1\!/_K$, and $\gamma_2, \ldots, \gamma_{K-1}$ evenly spaced (in dB) between γ_1 and γ_K



Minimum Expected Capacity Loss



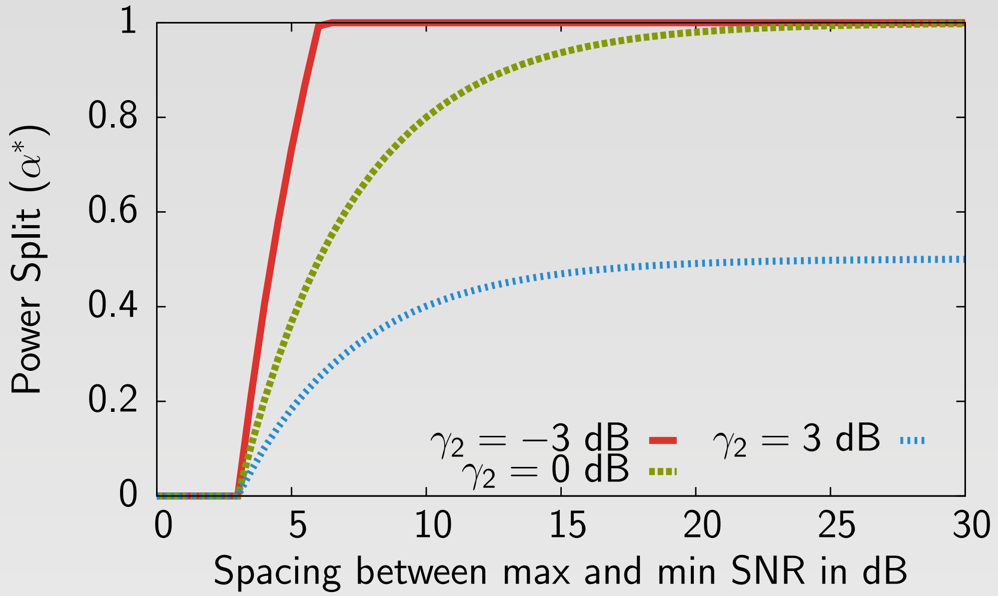
Minimum Fractional Expected Capacity Loss

- $J_{\mathbb{E},|\cdot|}(\boldsymbol{\alpha}^*)$ is increasing in γ_1
- For small γ_1, $J_{\mathbb{E},|\cdot|}(\boldsymbol{\alpha}^*)$ looks to be linear and independent of K.

- $J_{\%,\mathbb{E}}(\boldsymbol{\alpha}^*)$ is *not* monotonic in γ_1.
- For small γ_1, $J_{\%,\mathbb{E}}(\boldsymbol{\alpha}^*)$ looks to be linear and independent of K.

Two-State Channel Results

Optimal Power Split

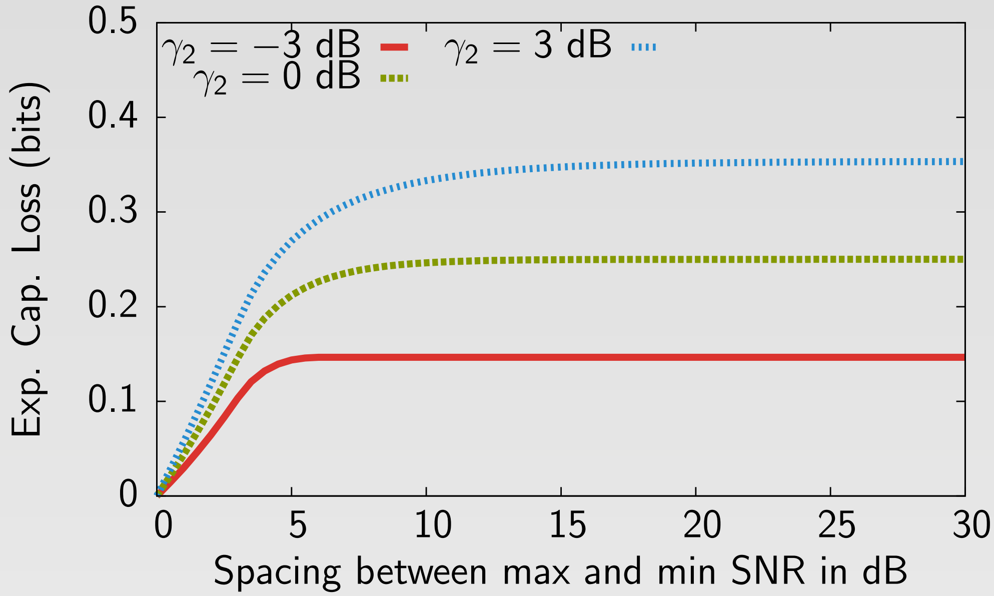


The optimal power split α^*

- is identically 0 for $\delta < {}^1/_{p_1} = 3$ dB
- is monotonically increasing in δ
- approaches the limits
  - 1 at $\delta = 6$ dB for $\gamma_2 = -3$ dB
  - 1 for $\gamma_2 = 0$ dB
  - 0.5 for $\gamma_2 = 3$ dB

Two-State Channel Results

Minimum Expected Capacity Loss

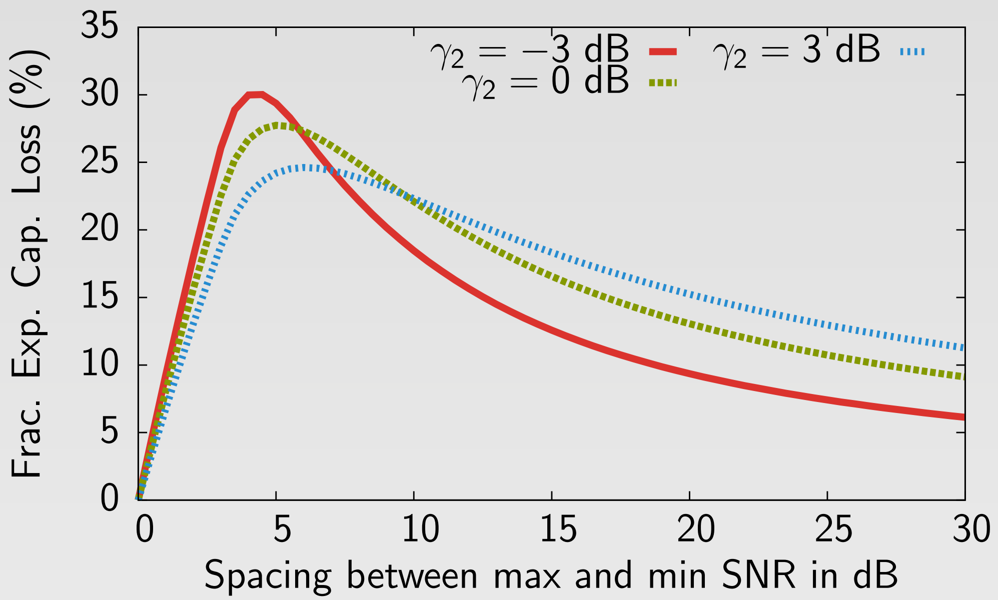


The minimum expected capacity loss $J_{\mathbb{E},|\cdot|}(\boldsymbol{\alpha}^*)$

- is equal to 0 for $\delta = 1$ (0 dB)
- is monotonically increasing in δ
- bounded by
  - 0.1465 bits for $\gamma_2 = -3$ dB,
  - 0.2500 bits for $\gamma_2 = 0$ dB, and
  - 0.3535 bits for $\gamma_2 = 3$ dB

Two-State Channel Results

Minimum Fractional Capacity Loss



The minimum fractional expected capacity loss $J_{\%,\mathbb{E}}(\boldsymbol{\alpha}^*)$

- is increasing in δ over the interval 0 dB to 3 dB
- is non-monotonic over the full range of δ
- converges rather slowly to 0.

Conclusions & Future Work

- K-user broadcast channel can model a point-to-point channel with K unknown states
- Using broadcast model to measure the loss in performance as compared to an omniscient transmitter
- Demonstrated several non-obvious properties of the optimal power split for $K = 2$ states

- $J_{\mathbb{E},|\cdot|}(\boldsymbol{\alpha}^*)$ and $J_{\%,\mathbb{E}}(\boldsymbol{\alpha}^*)$ approach a limiting function
- Use calculus of variations to characterize the limiting function as $K \to \infty$
  - The case of Rayleigh distributed channel fades in [3]

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